**Modification of Maxwell’s Equations: A Proca Field Theory Extension Based on Non-Zero Photon Rest Mass**

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**Abstract:**Based on the physical fact that the experimental upper limit on the photon rest mass () is non-zero, this paper proposes a fundamental modification to the classical Maxwell’s equations. The core argument is: Maxwell’s equations are a zero-mass approximation for the case where the photon rest mass ; their more general form is the Proca equations, which describe photons with non-zero rest mass. By introducing a residual mass term derived from Li Zhijun’s ABC field combination theory, where we construct a modified electromagnetic field Lagrangian. This modification term is directly linked to the residual coupling energy of the C-field (Higgs field) within the photon’s field-combination state, i.e., The modified equations naturally predict the existence of a photon rest mass and perfectly reduce to the classical Maxwell’s equations in the limit This work provides a new framework for unifying the mathematical form of electromagnetic theory with the physical reality of quantum field theory.

**Keywords:** Modification of Maxwell’s equations; Proca equation; Photon rest mass; ABC field combination theory; Gauge symmetry breaking

1. **Introduction: From Perfect Symmetry to Physical Reality**

Classical Maxwell’s equations are built upon the assumption implying perfect U(1) gauge symmetry for the electromagnetic field. However, any physical theory should be an effective approximation of a more fundamental theory at some energy scale. The experimental upper limit on the photon rest mass suggests that electromagnetic theory might contain a tiny, yet non-zero, symmetry-breaking term. Based on Li Zhijun’s ABC field combination theory, the infinitely small but non-zero C-field component within the photon state provides a natural physical origin for this symmetry breaking.

1. **Theoretical Model: Modified Lagrangian and Equations of Motion**

**2.1 Modified Electromagnetic Field Lagrangian**

The most general relativistic Lagrangian for the electromagnetic field is the Proca Lagrangian, which takes the form:

Here, the key modification term is the mass term Where:  
\* is the electromagnetic field strength tensor.  
\* is the photon’s Compton wavenumber; its reciprocal is the Compton wavelength, characterizing the quantum mechanical scale of a massive photon.  
\* This term directly breaks the U(1) gauge symmetry as the mass term is not invariant under gauge transformation.

**2.2 Modified Maxwell’s Equations (Proca Equations)**

Applying the Euler-Lagrange equations to the above Lagrangian yields the modified Maxwell’s equations:

1. Inhomogeneous Equations (with sources)
2. Homogeneous Equations (source-free) (Remain unchanged)
3. Lorenz Condition: Due to the presence of the mass term, the Lorenz condition is no longer an arbitrary gauge choice but a constraint that must be satisfied, derived from the equations of motion:

This eliminates gauge freedom and determines the physical polarization states.

**2.3 Connection to the ABC Field Combination Theory**

In the ABC theory, the photon’s rest mass originates from its residual coupling to the C-field (Higgs field): Therefore, the Proca mass term can be written as:

This equation directly links the modification term to the intrinsic property of the photon’s field-combination state (the C-field energy level providing a profound physical interpretation for the Proca equation.

1. **Physical Implications and Observable Effects**

The modified equations predict several physical effects different from classical theory, which vanish when :

1. Yukawa Potential for Electrostatics: The potential generated by a stationary point charge satisfies , whose solution is the Yukawa potential:

This implies that the electrostatic field experiences exponential attenuation at large distances, requiring a modification of Coulomb’s law.

1. Dispersion Relation in Vacuum: The dispersion relation for electromagnetic wave propagation in vacuum becomes:

This means light waves of different frequencies have different phase velocities in vacuum, although the group velocity remains the speed of information transfer. This could lead to observable dispersion effects for starlight over long propagation distances.

1. Additional Polarization State: Because the Lorenz condition becomes mandatory, the photon transitions from a transverse wave (2 polarization states) to a massive vector particle with 3 independent polarization states (2 transverse + 1 longitudinal).
2. **Conclusion: Significance of the Modification and the Status of Classical Theory**

By introducing a mass term related to the intrinsic properties of the photon’s field-combination state, this paper successfully modifies Maxwell’s equations into the more general Proca form:  
1. Mathematical Rigor: The modified equation set is self-consistent, relativistic, and incorporates the physical possibility of a non-zero photon rest mass.  
2. Physical Profundity: It deeply unifies the mathematical formulation of electromagnetic theory with the physical picture of quantum field theory (ABC theory). The modification term is no longer an ad hoc addition but physical evidence for the existence of the C-field component within the photon as a field-combination state.  
3. Status of Classical Theory: Given the extremely small value of (Compton wavelength light-years, far exceeding cosmic scales), the approximation remains exceptionally accurate for all macroscopic and most microscopic experiments conducted to date. Therefore, classical Maxwell’s equations continue to be a great and highly accurate effective theory for describing electromagnetic interactions.

This modification model provides a rigorous theoretical framework for testing the photon rest mass in ultra-high precision experiments or on cosmological scales. It is also an important application of the ABC field combination theory, establishing a strict theoretical basis for exploring the photon rest mass.

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